Package 'mnt'

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Type Package

Title Affine Invariant Tests of Multivariate Normality

Version 1.3

Description Various affine invariant multivariate normality tests are provided. It is designed to accompany the survey article Ebner, B. and Henze, N. (2020) <arXiv:2004.07332> titled ``Tests for multivariate normality -- a critical review with emphasis on weighted L^2-statistics''. We implement new and time honoured L^2-type tests of multivariate normality, such as the Baringhaus-Henze-Epps-Pulley (BHEP) test, the Henze-Zirkler test, the test of Henze-Jiménes-Gamero, the test of Henze-Jiménes-Gamero-Meintanis, the test of Henze-Visage, the Dörr-Ebner-Henze test based on harmonic oscillator and the Dörr-Ebner-Henze test based on a double estimation in a PDE. Secondly, we include the measures of multivariate skewness and kurtosis by Mardia, Koziol, Malkovich and Afifi and Móri, Rohatgi and Székely, as well as the associated tests. Thirdly, we include the tests of multivariate normality by Cox and Small, the 'energy' test of Székely and Rizzo, the tests based on spherical harmonics by Manzotti and Quiroz and the test of Pudelko. All the functions and tests need the data to be a n x d matrix where n is the samplesize (number of rows) and d is the dimension (number of columns).

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BHEP *Statistic of the BHEP-test*

Description

This function returns the value of the statistic of the Baringhaus-Henze-Epps-Pulley (BHEP) test as in Henze and Wagner (1997).

Usage

```
BHEP(data, a = 1)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`a`	positive numeric number (tuning parameter).

Details

The test statistic is

$$BHEP_{n,\beta} = \frac{1}{n}\sum_{j,k=1}^{n} \exp\left(-\frac{\beta^2\|Y_{n,j}-Y_{n,k}\|^2}{2}\right) - \frac{2}{(1+\beta^2)^{d/2}}\sum_{j=1}^{n}\exp\left(-\frac{\beta^2\|Y_{n,j}\|^2}{2(1+\beta^2)}\right) + \frac{n}{(1+2\beta^2)^{d/2}}.$$

Here, $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $j = 1,\ldots,n$, are the scaled residuals, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1,\ldots,X_n$. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the function returns an error.

Value

value of the test statistic.

References

Henze, N., and Wagner, T. (1997), A new approach to the class of BHEP tests for multivariate normality, J. Multiv. Anal., 62:1–23, DOI

Epps T.W., Pulley L.B. (1983), A test for normality based on the empirical characteristic function, Biometrika, 70:723-726, DOI

Examples

```
BHEP(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

CS *Statistic of the test of Cox and Small*

Description

This function returns the (approximated) value of the test statistic of the test of Cox and Small (1978).

Usage

```
CS(data, Points = NULL)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`Points`	points for approximation of the maximum on the sphere. `Points=NULL` generates 5000 uniformly distributed Points on the d dimensional unit sphere.

Details

The test statistic is $T_{n,CS} = \max_{b\in\{x\in\mathbf{R}^d:\|x\|=1\}} \eta_n^2(b)$, where

$$\eta_n^2(b) = \frac{\left\|n^{-1}\sum_{j=1}^n Y_{n,j}(b^\top Y_{n,j})^2\right\|^2 - \left(n^{-1}\sum_{j=1}^n (b^\top Y_{n,j})^3\right)^2}{n^{-1}\sum_{j=1}^n (b^\top Y_{n,j})^4 - 1 - \left(n^{-1}\sum_{j=1}^n (b^\top Y_{n,j})^3\right)^2}$$

. Here, $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $j = 1, \ldots, n$, are the scaled residuals, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the function returns an error. Note that the maximum functional has to be approximated by a discrete version, for details see Ebner (2012).

Value

approximation of the value of the test statistic of the test of Cox and Small (1978).

References

Cox, D.R. and Small, N.J.H. (1978), Testing multivariate normality, Biometrika, 65:263–272.

Ebner, B. (2012), Asymptotic theory for the test for multivariate normality by Cox and Small, Journal of Multivariate Analysis, 111:368–379.

Examples

```
CS(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

cv.quan *Monte Carlo simulation of quantiles for normality tests*

Description

This function returns the quantiles of a test statistic with optional tuning parameter.

Usage

```
cv.quan(
  samplesize,
  dimension,
  quantile,
  statistic,
  tuning = NULL,
  repetitions = 1e+05
)
```

Arguments

samplesize	samplesize for which the empirical quantile should be calculated.
dimension	a natural number to specify the dimension of the multivariate normal distribution
quantile	a number between 0 and 1 to specify the quantile of the empirical distribution of the considered test
statistic	a function specifying the test statistic.
tuning	the tuning parameter of the test statistic.
repetitions	number of Monte Carlo runs.

Value

empirical quantile of the test statistic.

Examples

```
cv.quan(samplesize=10, dimension=2,quantile=0.95, statistic=BHEP, tuning=2.5, repetitions=1000)
```

DEHT *Statistic of the DEH test based on harmonic oscillator*

Description

Computes the test statistic of the DEH test.

Usage

```
DEHT(data, a = 1)
```

Arguments

`data`	a n x d numeric matrix of data values.
`a`	positive numeric number (tuning parameter).

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter `a`. Each row of the data Matrix contains one of the `n` (multivariate) sample with dimension `d`. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

The value of the test statistic.

References

Dörr, P., Ebner, B., Henze, N. (2019) "Testing multivariate normality by zeros of the harmonic oscillator in characteristic function spaces" arXiv:1909.12624

Examples

```
DEHT(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=1)
```

DEHU *Statistic of the DEH test based on a double estimation in PDE*

Description

Computes the test statistic of the DEH based on a double estimation in PDE test.

Usage

```
DEHU(data, a)
```

Arguments

`data`	a (d,n) numeric matrix containing the data.
`a`	positive numeric number (tuning parameter).

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter `a`. Each row of the data Matrix contains one of the `n` (multivariate) sample with dimension `d`. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

The value of the test statistic.

References

Dörr, P., Ebner, B., Henze, N. (2019) "A new test of multivariate normality by a double estimation in a characterizing PDE" arXiv:1911.10955

EHS *Statistic of the EHS test based on a multivariate Stein equation*

Description

Computes the test statistic of the EHS test based on a multivariate Stein equation.

Usage

```
EHS(data, a = 1)
```

Arguments

`data`	a (d,n) numeric matrix containing the data.
`a`	positive numeric number (tuning parameter).

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter a. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Note that a=Inf returns the limiting test statistic with value 2*MSkew + MRSSkew and a=0 returns the value of the limit statistic

$$T_{n,0} = \frac{d}{2} - 2^{\frac{d}{2}+1} \frac{1}{n} \sum_{j=1}^{n} \|Y_{n,j}\|^2 \exp(-\frac{\|Y_{n,j}\|^2}{2}).$$

Value

The value of the test statistic.

References

Ebner, B., Henze, N., Strieder, D. (2020) "Testing normality in any dimension by Fourier methods in a multivariate Stein equation" arXiv:2007.02596

HJG *Henze-Jiménes-Gamero test statistic*

Description

Computes the test statistic of the Henze-Jimenes-Gamero test.

Usage

```
HJG(data, a = 5)
```

Arguments

data	a n x d numeric matrix of data values.
a	positive numeric number (tuning parameter).

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter a. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the function returns an error.

Value

The value of the test statistic.

References

Henze, N., Jiménez-Gamero, M.D. (2019) "A new class of tests for multinormality with i.i.d. and garch data based on the empirical moment generating function", TEST, 28, 499-521, DOI

Examples

```
HJG(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=5)
```

HJM — *statistic of the Henze-Jiménes-Gamero-Meintanis test*

Description

Computes the test statistic of the Henze-Jiménes-Gamero-Meintanis test.

Usage

```
HJM(data, a)
```

Arguments

`data`	a n x d numeric matrix of data values.
`a`	positive numeric number (tuning parameter).

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter `a`. Each row of the data Matrix contains one of the `n` (multivariate) sample with dimension `d`. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the function returns an error.

Value

The value of the test statistic.

References

Henze, N., Jiménes-Gamero, M.D., Meintanis, S.G. (2019), Characterizations of multinormality and corresponding tests of fit, including for GARCH models, Econometric Th., 35:510–546, DOI.

Examples

```
HJM(MASS::mvrnorm(20,c(0,1),diag(1,2)),a=2.5)
```

HV *statistic of the Henze-Visagie test*

Description

Computes the test statistic of the Henze-Visagie test.

Usage

```
HV(data, a = 5)
```

Arguments

`data`	a n x d numeric matrix of data values.
`a`	numeric number greater than 1 (tuning parameter).

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter `a`. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the function returns an error.

Note that `a=Inf` returns the limiting test statistic with value `2*MSkew + MRSSkew`.

Value

The value of the test statistic.

References

Henze, N., Visagie, J. (2019) "Testing for normality in any dimension based on a partial differential equation involving the moment generating function", to appear in Ann. Inst. Stat. Math., DOI

Examples

```
HV(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=5)
HV(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=Inf)
```

HZ *Statistic of the Henze-Zirkler test*

Description

This function returns the value of the statistic of the BHEP test as in Henze and Zirkler (1990). The difference to the BHEP test is in the choice of the tuning parameter β.

Usage

```
HZ(data)
```

Arguments

data	a n x d matrix of d dimensional data vectors.

Details

A BHEP test is performed with tuning parameter β chosen in dependence of the sample size n and the dimension d, namely

$$\beta = \frac{((2d+1)n/4)^(1/(d+4))}{\sqrt{2}}.$$

Value

value of the test statistic.

References

Henze, N., and Zirkler, B. (1990), A class of invariant consistent tests for multivariate normality, Commun.-Statist. – Th. Meth., 19:3595–3617, DOI

See Also

BHEP

Examples

```
HZ(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

KKurt *Koziols measure of multivariate sample kurtosis*

Description

This function computes the invariant measure of multivariate sample kurtosis due to Koziol (1989).

Usage

```
KKurt(data)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.

Details

Multivariate sample kurtosis due to Koziol (1989) is defined by

$$\widetilde{b}^{(2)}_{n,d} = \frac{1}{n^2} \sum_{j,k=1}^{n} (Y_{n,j}^{\top} Y_{n,k})^4,$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $j = 1, \ldots, n$, are the scaled residuals, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the function returns an error. Note that for $d = 1$, we have a measure proportional to the squared sample kurtosis.

Value

value of sample kurtosis in the sense of Koziol.

References

Koziol, J.A. (1989), A note on measures of multivariate kurtosis, Biom. J., 31:619–624.

Examples

```
KKurt(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

MAKurt *multivariate kurtosis in the sense of Malkovich and Afifi*

Description

This function computes the invariant measure of multivariate sample kurtosis due to Malkovich and Afifi (1973).

Usage

```
MAKurt(data, Points = NULL)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`Points`	points for approximation of the maximum on the sphere. `Points=NULL` generates 1000 uniformly distributed Points on the d dimensional unit sphere.

Details

Multivariate sample skewness due to Malkovich and Afifi (1973) is defined by

$$b_{n,d,M}^{(1)} = \max_{u\in\{x\in\mathbf{R}^d:\|x\|=1\}} \frac{\left(\frac{1}{n}\sum_{j=1}^n (u^\top X_j - u^\top \overline{X}_n)^3\right)^2}{(u^\top S_n u)^3},$$

where $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \dots, X_n$. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the function returns an error.

Value

value of sample kurtosis in the sense of Malkovich and Afifi.

References

Malkovich, J.F., and Afifi, A.A. (1973), On tests for multivariate normality, J. Amer. Statist. Ass., 68:176–179.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467–506.

Examples

```
MAKurt(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

MASkew *multivariate skewness in the sense of Malkovich and Afifi*

Description

This function computes the invariant measure of multivariate sample skewness due to Malkovich and Afifi (1973).

Usage

```
MASkew(data, Points = NULL)
```

Arguments

data	a n x d matrix of d dimensional data vectors.
Points	points for approximation of the maximum on the sphere. `Points=NULL` generates 1000 uniformly distributed Points on the d dimensional unit sphere.

Details

Multivariate sample skewness due to Malkovich and Afifi (1973) is defined by

$$b_{n,d,M}^{(1)} = \max_{u \in \{x \in \mathbf{R}^d : \|x\|=1\}} \frac{\left(\frac{1}{n}\sum_{j=1}^n (u^\top X_j - u^\top \overline{X}_n)^3\right)^2}{(u^\top S_n u)^3},$$

where $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the function returns an error.

Value

value of sample skewness in the sense of Malkovich and Afifi.

References

Malkovich, J.F., and Afifi, A.A. (1973), On tests for multivariate normality, J. Amer. Statist. Ass., 68:176–179.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467–506.

Examples

```
MASkew(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

MKurt *Mardias measure of multivariate sample kurtosis*

Description

This function computes the classical invariant measure of multivariate sample kurtosis due to Mardia (1970).

Usage

```
MKurt(data)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.

Details

Multivariate sample kurtosis due to Mardia (1970) is defined by

$$b_{n,d}^{(2)} = \frac{1}{n}\sum_{j=1}^{n} \|Y_{n,j}\|^4,$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$.To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the function returns an error.

Value

value of sample kurtosis in the sense of Mardia.

References

Mardia, K.V. (1970), Measures of multivariate skewness and kurtosis with applications, Biometrika, 57:519–530.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467–506.

Examples

```
MKurt(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

MQ1 *first statistic of Manzotti and Quiroz*

Description

This function returns the value of the first statistic of Manzotti and Quiroz (2001).

Usage

```
MQ1(data)
```

Arguments

data	a n x d matrix of d dimensional data vectors.

Value

Value of the test statistic

References

Manzotti, A., and Quiroz, A.J. (2001), Spherical harmonics in quadratic forms for testing multivariate normality, Test, 10:87–104, DOI

Examples

```
MQ1(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

MQ2 *second statistic of Manzotti und Quiroz*

Description

This function returns the value of the second statistic of Manzotti und Quiroz (2001).

Usage

```
MQ2(data)
```

Arguments

data	a n x d matrix of d dimensional data vectors.

Value

Value of the test statistic

References

Manzotti, A., and Quiroz, A.J. (2001), Spherical harmonics in quadratic forms for testing multivariate normality, Test, 10:87–104, DOI

Examples

```
MQ2(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

MRSSkew — *multivariate skewness of Móri, Rohatgi and Székely*

Description

This function computes the invariant measure of multivariate sample skewness due to Móri, Rohatgi and Székely (1993).

Usage

```
MRSSkew(data)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.

Details

Multivariate sample skewness due to Móri, Rohatgi and Székely (1993) is defined by

$$\widetilde{b}_{n,d}^{(1)} = \frac{1}{n}\sum_{j=1}^{n} \|Y_{n,j}\|^2 \|Y_{n,k}\|^2 Y_{n,j}^\top Y_{n,k},$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the function returns an error. Note that for $d = 1$, it is equivalent to skewness in the sense of Mardia.

Value

value of sample skewness in the sense of Móri, Rohatgi and Székely.

References

Móri, T. F., Rohatgi, V. K., Székely, G. J. (1993), On multivariate skewness and kurtosis, Theory of Probability and its Applications, 38:547–551.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467–506.

MSkew *Mardias measure of multivariate sample skewness*

Description

This function computes the classical invariant measure of multivariate sample skewness due to Mardia (1970).

Usage

```
MSkew(data)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.

Details

Multivariate sample skewness due to Mardia (1970) is defined by

$$b_{n,d}^{(1)} = \frac{1}{n^2} \sum_{j,k=1}^{n} (Y_{n,j}^\top Y_{n,k})^3,$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the function returns an error. Note that for $d = 1$, we have a measure proportional to the squared sample skewness.

Value

value of sample skewness in the sense of Mardia.

References

Mardia, K.V. (1970), Measures of multivariate skewness and kurtosis with applications, Biometrika, 57:519–530.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467–506.

Examples

```
MSkew(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

print.mnt — *Print method for tests of multivariate normality*

Description

Printing objects of class "mnt".

Usage

```
## S3 method for class 'mnt'
print(x, ...)
```

Arguments

`x`	object of class "mnt".
`...`	further arguments to be passed to or from methods.

Details

A `mnt` object is a named list of numbers and character string, supplemented with `test` (the name of the teststatistic). `test` is displayed as a title. The remaining elements are given in an aligned "name = value" format.

Value

the argument x, invisibly, as for all print methods.

Examples

```
print(test.DEHU(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=1,MC=500))
```

PU — *Statistic of the Pudelko test*

Description

Approximates the test statistic of the Pudelko test.

Usage

```
PU(data, r = 2)
```

Arguments

`data`	a n x d numeric matrix of data values.
`r`	a positive number (radius of Ball)

Details

This functions evaluates the test statistic with the given data and the specified parameter `r`. Since since one has to calculate the supremum of a function inside a d-dimensional Ball of radius `r`. In this implementation the `optim` function is used.

Value

approximate Value of the test statistic

References

Pudelko, J. (2005), On a new affine invariant and consistent test for multivariate normality, Probab. Math. Statist., 25:43–54.

Examples

```
PU(MASS::mvrnorm(20,c(0,1),diag(1,2)),r=2)
```

Quantile09 — *Simulated empirical 90% quantiles of the tests contained in package* `mnt`

Description

A dataset containing the empirical 0.9 quantiles of the tests for the dimensions `d=2,3,5` and samplesizes `n=20,50,100` based on a Monte Carlo Simulation study with 100000 repetitions. The following parameters were used:

- For `BHEP` the parameter `a=1`,
- for `HV` the parameter `a=5`,
- for `HJG` the parameter `a=1.5`,
- for `HJM` the parameter `a=1.5`,
- for `DEHT` the parameter `a=0.25`,
- for `DEHU` the parameter `a=0.5`,
- for `CS` the parameter `Points=NULL`,
- for `PU` the parameter `r=2`,
- for `MASkew` the parameter `Points=NULL`,
- for `MAKurt` the parameter `Points=NULL`,

Usage

```
Quantile09
```

Format

A data frame with 9 rows and 20 columns.

Quantile095 — *Simulated empirical 95% quantiles of the tests contained in package* mnt

Description

A dataset containing the empirical 0.95 quantiles of the tests for the dimensions d=2,3,5 and samplesizes n=20,50,100 based on a Monte Carlo Simulation study with 100000 repetitions. The following parameters were used:

- For BHEP the parameter a=1,
- for HV the parameter a=5,
- for HJG the parameter a=1.5,
- for HJM the parameter a=1.5,
- for DEHT the parameter a=0.25,
- for DEHU the parameter a=0.5,
- for CS the parameter Points=NULL,
- for PU the parameter r=2,
- for MASkew the parameter Points=NULL,
- for MAKurt the parameter Points=NULL,

Usage

```
Quantile095
```

Format

A data frame with 9 rows and 20 columns.

Quantile099 — *Simulated empirical 99% quantiles of the tests contained in package* mnt

Description

A dataset containing the empirical 0.99 quantiles of the tests for the dimensions d=2,3,5 and samplesizes n=20,50,100 based on a Monte Carlo Simulation study with 100000 repetitions. The following parameters were used:

- For BHEP the parameter a=1,
- for HV the parameter a=5,
- for HJG the parameter a=1.5,

- for `HJM` the parameter `a=1.5`,
- for `DEHT` the parameter `a=0.25`,
- for `DEHU` the parameter `a=0.5`,
- for `CS` the parameter `Points=NULL`,
- for `PU` the parameter `r=2`,
- for `MASkew` the parameter `Points=NULL`,
- for `MAKurt` the parameter `Points=NULL`,

Usage

```
Quantile099
```

Format

A data frame with 9 rows and 20 columns.

SR *statistic of the Székely-Rizzo test*

Description

This function returns the value of the statistic of the test of multivariate normality (also called *energy test*) as in Székely and Rizzo (2005). Note that the scaled residuals use another scaling in the estimator of the covariance matrix as the other functions of the package `mnt`! It is equivalent to the function `mvnorm.e`.

Usage

```
SR(data, abb = 1e-08)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`abb`	Stop criterium.

Value

value of the test statistic.

References

Székely, G., and Rizzo, M. (2005), A new test for multivariate normality, J. Multiv. Anal., 93:58–80, DOI

See Also

`mvnorm.e`

Examples

```
SR(MASS::mvrnorm(50,c(0,1),diag(1,2)))
```

`standard` *Empirical scaled residuals*

Description

A function that computes the scaled residuals of the data.

Usage

```
standard(data)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors..

Value

A n x d matrix of the scaled residuals.

`test.BHEP` *Baringhaus-Henze-Epps-Pulley (BHEP) test*

Description

Performs the BHEP test of multivariate normality as suggested in Henze and Wagner (1997) using a tuning parameter `a`.

Usage

```
test.BHEP(data, a = 1, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`a`	positive numeric number (tuning parameter).
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Details

The test statistic is

$$BHEP_{n,\beta} = \frac{1}{n}\sum_{j,k=1}^{n}\exp\left(-\frac{\beta^2\|Y_{n,j}-Y_{n,k}\|^2}{2}\right) - \frac{2}{(1+\beta^2)^{d/2}}\sum_{j=1}^{n}\exp\left(-\frac{\beta^2\|Y_{n,j}\|^2}{2(1+\beta^2)}\right) + \frac{n}{(1+2\beta^2)^{d/2}}.$$

Here, $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $j = 1, \ldots, n$, are the scaled residuals, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.#'

References

Henze, N., Wagner, T. (1997), A new approach to the class of BHEP tests for multivariate normality, J. Multiv. Anal., 62:1-23, DOI

See Also

BHEP

Examples

```
test.BHEP(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

---

test.CS *multivariate normality test of Cox and Small*

Description

Performs the test of multivariate normality of Cox and Small (1978).

Usage

```
test.CS(data, MC.rep = 1000, alpha = 0.05, Points = NULL)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value.
`alpha`	level of significance of the test.
`Points`	number of points to approximate the maximum functional on the unit sphere.

Details

The test statistic is $T_{n,CS} = \max_{b \in \{x \in \mathbf{R}^d : \|x\|=1\}} \eta_n^2(b)$, where

$$\eta_n^2(b) = \frac{\left\| n^{-1} \sum_{j=1}^n Y_{n,j}(b^\top Y_{n,j})^2 \right\|^2 - \left(n^{-1} \sum_{j=1}^n (b^\top Y_{n,j})^3 \right)^2}{n^{-1} \sum_{j=1}^n (b^\top Y_{n,j})^4 - 1 - \left(n^{-1} \sum_{j=1}^n (b^\top Y_{n,j})^3 \right)^2}$$

. Here, $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $j = 1, \ldots, n$, are the scaled residuals, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error. Note that the maximum functional has to be approximated by a discrete version, for details see Ebner (2012).

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.#'

References

Cox, D.R., Small, N.J.H. (1978), Testing multivariate normality, Biometrika, 65:263-272.

Ebner, B. (2012), Asymptotic theory for the test for multivariate normality by Cox and Small, Journal of Multivariate Analysis, 111:368-379.

See Also

`CS`

Examples

```
test.CS(MASS::mvrnorm(10,c(0,1),diag(1,2)),MC.rep=100)
```

`test.DEHT` *Doerr-Ebner-Henze test of multivariate normality based on harmonic oscillator*

Description

Computes the multivariate normality test of Doerr, Ebner and Henze (2019) based on zeros of the harmonic oscillator.

Usage

```
test.DEHT(data, a = 1, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`a`	positive numeric number (tuning parameter).
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value.
`alpha`	level of significance of the test.

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter `a`. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Doerr, P., Ebner, B., Henze, N. (2019) "Testing multivariate normality by zeros of the harmonic oscillator in characteristic function spaces" arXiv:1909.12624

See Also

DEHT

Examples

```
test.DEHT(MASS::mvrnorm(20,c(0,1),diag(1,2)),a=1,MC=500)
```

`test.DEHU` *Doerr-Ebner-Henze test of multivariate normality based on a double estimation in a PDE*

Description

Computes the multivariate normality test of Doerr, Ebner and Henze (2019) based on a double estimation in a PDE.

Usage

```
test.DEHU(data, a = 0.5, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`a`	positive numeric number (tuning parameter).
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value.
`alpha`	level of significance of the test.

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter `a`. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Doerr, P., Ebner, B., Henze, N. (2019) "Testing multivariate normality by zeros of the harmonic oscillator in characteristic function spaces" arXiv:1909.12624

See Also

DEHU

Examples

```
test.DEHU(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=1,MC=500)
```

`test.EHS` *Ebner-Henze-Strieder test of multivariate normality based on Fourier methods in a multivariate Stein equation*

Description

Computes the multivariate normality test of Ebner, Henze and Strieder (2020) based on Fourier methods in a multivariate Stein equation.

Usage

```
test.EHS(data, a = 0.5, MC.rep = 10000, alpha = 0.05)
```

Arguments

data	a n x d matrix of d dimensional data vectors.
a	positive numeric number (tuning parameter).
MC.rep	number of repetitions for the Monte Carlo simulation of the critical value.
alpha	level of significance of the test.

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter a. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level alpha:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Ebner, B., Henze, N., Strieder, D. (2020) "Testing normality in any dimension by Fourier methods in a multivariate Stein equation" arXiv:2007.02596

See Also

EHS

Examples

```
test.EHS(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=1,MC=500)
```

---

test.HJG — *Henze-Jimenes-Gamero test of multivariate normality*

Description

Computes the multivariate normality test of Henze and Jimenes-Gamero (2019) in dependence of a tuning parameter `a`.

Usage

```
test.HJG(data, a = 1, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`a`	positive numeric number (tuning parameter).
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value.
`alpha`	level of significance of the test.

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter `a`. Each row of the data Matrix contains one of the `n` (multivariate) sample with dimension `d`. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.#'

References

Henze, N., Jimenez-Gamero, M.D. (2019) "A new class of tests for multinormality with i.i.d. and garch data based on the empirical moment generating function", TEST, 28, 499-521, DOI

See Also

HJG

Examples

```
test.HJG(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=1.5,MC.rep=500)
```

---

test.HJM — *Henze-Jimenes-Gamero-Meintanis test of multivariate normality*

Description

Computes the test statistic of the Henze-Jimenes-Gamero-Meintanis test.

Usage

```
test.HJM(data, a = 1.5, MC.rep = 500, alpha = 0.05)
```

Arguments

data	a n x d matrix of d dimensional data vectors.
a	positive numeric number (tuning parameter).
MC.rep	number of repetitions for the Monte Carlo simulation of the critical value.
alpha	level of significance of the test.

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter a. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Henze, N., Jimenes-Gamero, M.D., Meintanis, S.G. (2019), Characterizations of multinormality and corresponding tests of fit, including for GARCH models, Econometric Th., 35:510-546, DOI.

See Also

HJM

Examples

```
test.HJM(MASS::mvrnorm(10,c(0,1),diag(1,2)),a=2.5,MC=100)
```

`test.HV` *The Henze-Visagie test of multivariate normality*

Description

Computes the multivariate normality test of Henze and Visagie (2019).

Usage

```
test.HV(data, a = 5, MC.rep = 10000, alpha = 0.05)
```

Arguments

data	a n x d matrix of d dimensional data vectors.
a	positive numeric number (tuning parameter).
MC.rep	number of repetitions for the Monte Carlo simulation of the critical value.
alpha	level of significance of the test.

Details

This functions evaluates the teststatistic with the given data and the specified tuning parameter a. Each row of the data Matrix contains one of the n (multivariate) sample with dimension d. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Note that a=Inf returns the limiting test statistic with value 2*MSkew + MRSSkew.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level alpha:

$Test name of the test.

$param value tuning parameter.

$Test.value the value of the test statistic.

$cv the approximated critical value.

$Decision the comparison of the critical value and the value of the test statistic.

References

Henze, N., Visagie, J. (2019) "Testing for normality in any dimension based on a partial differential equation involving the moment generating function", to appear in Ann. Inst. Stat. Math., DOI

See Also

HV

Examples

```
test.HV(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=5,MC.rep=500)
test.HV(MASS::mvrnorm(50,c(0,1),diag(1,2)),a=Inf,MC.rep=500)
```

---

test.HZ *The Henze-Zirkler test*

Description

Performs the test of multivariate normality of Henze and Zirkler (1990).

Usage

```
test.HZ(data, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Details

A `BHEP` test is performed with tuning parameter β chosen in dependence of the sample size n and the dimension d, namely

$$\beta = \frac{((2d+1)n/4)^(1/(d+4))}{\sqrt{2}}.$$

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.#'

References

Henze, N., Zirkler, B. (1990), A class of invariant consistent tests for multivariate normality, Commun.-Statist. - Th. Meth., 19:3595-3617, DOI

See Also

`HZ`

Examples

```
test.HZ(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

`test.KKurt` *Test of normality based on Koziols measure of multivariate sample kurtosis*

Description

Computes the multivariate normality test based on the invariant measure of multivariate sample kurtosis due to Koziol (1989).

Usage

```
test.KKurt(data, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Details

Multivariate sample kurtosis due to Koziol (1989) is defined by

$$\widetilde{b}_{n,d}^{(2)} = \frac{1}{n^2}\sum_{j,k=1}^{n}(Y_{n,j}^{\top}Y_{n,k})^4,$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $j = 1, \ldots, n$, are the scaled residuals, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error. Note that for $d = 1$, we have a measure proportional to the squared sample kurtosis.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Koziol, J.A. (1989), A note on measures of multivariate kurtosis, Biom. J., 31:619-624.

See Also

KKurt

Examples

```
test.KKurt(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

`test.MAKurt` *Test of normality based on multivariate kurtosis in the sense of Malkovich and Afifi*

Description

Computes the multivariate normality test based on the invariant measure of multivariate sample kurtosis due to Malkovich and Afifi (1973).

Usage

```
test.MAKurt(data, MC.rep = 10000, alpha = 0.05, num.points = 1000)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test
`num.points`	number of points distributed uniformly over the sphere for approximation of the maximum on the sphere.

Details

Multivariate sample skewness due to Malkovich and Afifi (1973) is defined by

$$b_{n,d,M}^{(1)} = \max_{u \in \{x \in \mathbf{R}^d : \|x\|=1\}} \frac{\left(\frac{1}{n}\sum_{j=1}^{n}(u^\top X_j - u^\top \overline{X}_n)^3\right)^2}{(u^\top S_n u)^3},$$

where $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` number of points used in approximation.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Malkovich, J.F., and Afifi, A.A. (1973), On tests for multivariate normality, J. Amer. Statist. Ass., 68:176-179.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467-506.

See Also

`MAKurt`

Examples

```
test.MAKurt(MASS::mvrnorm(10,c(0,1),diag(1,2)),MC.rep=100)
```

---

test.MASkew — *Test of normality based on multivariate skewness in the sense of Malkovich and Afifi*

Description

Computes the test of multivariate normality based on skewness in the sense of Malkovich and Afifi (1973).

Usage

```
test.MASkew(data, MC.rep = 10000, alpha = 0.05, num.points = 1000)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test
`num.points`	number of points distributed uniformly over the sphere for approximation of the maximum on the sphere.

Details

Multivariate sample skewness due to Malkovich and Afifi (1973) is defined by

$$b^{(1)}_{n,d,M} = \max_{u \in \{x \in \mathbf{R}^d : \|x\|=1\}} \frac{\left(\frac{1}{n}\sum_{j=1}^{n}(u^\top X_j - u^\top \overline{X}_n)^3\right)^2}{(u^\top S_n u)^3},$$

where $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` number of points used in approximation.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Malkovich, J.F., and Afifi, A.A. (1973), On tests for multivariate normality, J. Amer. Statist. Ass., 68:176-179.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467-506.

See Also

`MASkew`

Examples

```
test.MASkew(MASS::mvrnorm(10,c(0,1),diag(1,2)),MC.rep=100)
```

`test.MKurt` *Test of normality based on Mardias measure of multivariate sample kurtosis*

Description

Computes the multivariate normality test based on the classical invariant measure of multivariate sample kurtosis due to Mardia (1970).

Usage

```
test.MKurt(data, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Details

Multivariate sample kurtosis due to Mardia (1970) is defined by

$$b_{n,d}^{(2)} = \frac{1}{n}\sum_{j=1}^{n} \|Y_{n,j}\|^4,$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$.To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Mardia, K.V. (1970), Measures of multivariate skewness and kurtosis with applications, Biometrika, 57:519-530.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467-506.

See Also

`MKurt`

Examples

```
test.MKurt(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

`test.MQ1` *Manzotti-Quiroz test 1*

Description

Performs the first test of multivariate normality of Manzotti and Quiroz (2001).

Usage

```
test.MQ1(data, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.#'

References

Manzotti, A., Quiroz, A.J. (2001), Spherical harmonics in quadratic forms for testing multivariate normality, Test, 10:87-104, DOI

See Also

`MQ1`

Examples

```
test.MQ1(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=100)
```

`test.MQ2` *Manzotti-Quiroz test 2*

Description

Performs the second test of multivariate normality of Manzotti and Quiroz (2001).

Usage

```
test.MQ2(data, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.#'

References

Manzotti, A., Quiroz, A.J. (2001), Spherical harmonics in quadratic forms for testing multivariate normality, Test, 10:87-104, DOI

See Also

MQ2

Examples

```
test.MQ2(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

`test.MRSSkew` *Test of multivariate normality based on the measure of multivariate skewness of Mori, Rohatgi and Szekely*

Description

Computes the multivariate normality test based on the invariant measure of multivariate sample skewness due to Mori, Rohatgi and Szekely (1993).

Usage

```
test.MRSSkew(data, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Details

Multivariate sample skewness due to Mori, Rohatgi and Szekely (1993) is defined by

$$\widetilde{b}_{n,d}^{(1)} = \frac{1}{n} \sum_{j=1}^{n} \|Y_{n,j}\|^2 \|Y_{n,k}\|^2 Y_{n,j}^\top Y_{n,k},$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d+1$ is needed. If that is not the case the test returns an error. Note that for $d = 1$, it is equivalent to skewness in the sense of Mardia.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Mori, T. F., Rohatgi, V. K., Szekely, G. J. (1993), On multivariate skewness and kurtosis, Theory of Probability and its Applications, 38:547-551.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467-506.

See Also

MRSSkew

Examples

```
test.MRSSkew(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

---

test.MSkew — *Test of normality based on Mardias measure of multivariate sample skewness*

Description

Computes the multivariate normality test based on the classical invariant measure of multivariate sample skewness due to Mardia (1970).

Usage

```
test.MSkew(data, MC.rep = 10000, alpha = 0.05)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value
`alpha`	level of significance of the test

Details

Multivariate sample skewness due to Mardia (1970) is defined by

$$b_{n,d}^{(1)} = \frac{1}{n^2} \sum_{j,k=1}^{n} (Y_{n,j}^\top Y_{n,k})^3,$$

where $Y_{n,j} = S_n^{-1/2}(X_j - \overline{X}_n)$, $\overline{X}_n$ is the sample mean and S_n is the sample covariance matrix of the random vectors $X_1, \ldots, X_n$. To ensure that the computation works properly $n \geq d + 1$ is needed. If that is not the case the test returns an error. Note that for $d = 1$, we have a measure proportional to the squared sample skewness.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Mardia, K.V. (1970), Measures of multivariate skewness and kurtosis with applications, Biometrika, 57:519-530.

Henze, N. (2002), Invariant tests for multivariate normality: a critical review, Statistical Papers, 43:467-506.

See Also

MSkew

Examples

```
test.MSkew(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

---

test.PU — *Pudelko test of multivariate normality*

Description

Computes the (approximated) Pudelko test of multivariate normality.

Usage

```
test.PU(data, MC.rep = 10000, alpha = 0.05, r = 2)
```

Arguments

`data`	a n x d matrix of d dimensional data vectors.
`MC.rep`	number of repetitions for the Monte Carlo simulation of the critical value.
`alpha`	level of significance of the test.
`r`	a positive number (radius of Ball)

Details

This functions evaluates the test statistic with the given data and the specified parameter `r`. Since since one has to calculate the supremum of a function inside a d-dimensional Ball of radius `r`. In this implementation the `optim` function is used.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$param` value tuning parameter.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.

References

Pudelko, J. (2005), On a new affine invariant and consistent test for multivariate normality, Probab. Math. Statist., 25:43-54.

See Also

PU

Examples

```
test.PU(MASS::mvrnorm(20,c(0,1),diag(1,2)),r=2,MC=100)
```

---

test.SR *Szekely-Rizzo (energy) test*

Description

Performs the test of multivariate normality of Szekely and Rizzo (2005). Note that the scaled residuals use another scaling in the estimator of the covariance matrix!

Usage

```
test.SR(data, MC.rep = 10000, alpha = 0.05, abb = 1e-08)
```

Arguments

data	a n x d matrix of d dimensional data vectors.
MC.rep	number of repetitions for the Monte Carlo simulation of the critical value
alpha	level of significance of the test
abb	Stop criterium.

Value

a list containing the value of the test statistic, the approximated critical value and a test decision on the significance level `alpha`:

`$Test` name of the test.

`$Test.value` the value of the test statistic.

`$cv` the approximated critical value.

`$Decision` the comparison of the critical value and the value of the test statistic.#'

References

Szekely, G., Rizzo, M. (2005), A new test for multivariate normality, J. Multiv. Anal., 93:58-80, DOI

See Also

SR

Examples

```
test.SR(MASS::mvrnorm(50,c(0,1),diag(1,2)),MC.rep=500)
```

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