Package ‘NLPwavelet’

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Type Package

Title Bayesian Wavelet Analysis Using Non-Local Priors

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Description Performs Bayesian wavelet analysis using individual non-local priors as described in Sanyal & Ferreira (2017) <DOI:10.1007/s13571-016-0129-3> and non-local prior mixtures as described in Sanyal (2025) <DOI:10.48550/arXiv.2501.18134>.

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Imports Rcpp (>= 1.0.14), wavethresh

LinkingTo Rcpp, RcppArmadillo

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BugReports https://github.com/nilotpalsanyal/NLPwavelet/issues

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Description

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Details

The main function is `BNLPWA`, which has arguments for specifying analysis using individual non-local priors or non-local prior mixtures and various hyperparameter specifications for the wavelet coefficients and scale parameters of the non-local priors. See the manual of `BNLPWA` for examples.

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References

Sanyal, Nilotpal. "Nonlocal prior mixture-based Bayesian wavelet regression." arXiv preprint arXiv:2501.18134 (2025).

Sanyal, Nilotpal, and Marco AR Ferreira. "Bayesian wavelet analysis using nonlocal priors with an application to FMRI analysis." Sankhya B 79.2 (2017): 361-388.

BNLPWA — *Bayesian Non-Local Prior-Based Wavelet Analysis*

Description

`BNLPWA` is the main function of this package that performs Bayesian wavelet analysis using individual non-local priors as described in Sanyal & Ferreira (2017) and non-local prior mixtures as described in Sanyal (2025). It currently works with one-dimensional data. The usage is described below.

Usage

```
BNLPWA(
  data,
  func=NULL,
  method=c("mixture","mom","imom"),
  mixprob_dist=c("logit","genlogit","hypsec","gennormal"),
  scale_dist=c("polynom","doubleexp"),
```

```
  r=1,
  nu=1,
  wave.family="DaubLeAsymm",
  filter.number=6,
  bc="periodic"
)
```

Arguments

`data`	Vector of noisy data.
`func`	Vector of true functional values. NULL by default. If available, they are used to compute and return mean squared error (MSE) of the estimates.
`method`	"mixture" for non-local prior mixture-based analysis, "mom" or "imom" for individual non-local prior-based analysis.
`mixprob_dist`	Specification for the mixture probabilities of the spike-and-slab prior. Equations given in the Details.
`scale_dist`	Specification for the scale parameters of the non-local priors. Equations given in the Details.
`r`	Integer specifying a) the order of the MOM prior or the shape parameter of the IMOM prior for individual non-local prior-based analysis, or b) the order of the MOM prior for non-local prior mixture-based analysis.
`nu`	Integer specifying the shape parameter of the IMOM prior for non-local prior mixture-based analysis. Not used for individual non-local prior-based analysis.
`wave.family`	The family of wavelets to use - "DaubExPhase" or "DaubLeAsymm". Default is "DaubLeAsymm".
`filter.number`	The number of vanishing moments of the wavelet. Default is 6.
`bc`	The boundary condition to use - "periodic" or "symmetric". Default is "periodic".

Details

Spike-and-slab prior for the wavelet coefficients:

For individual MOM prior-based analysis, the spike-and-slab prior for the wavelet coefficient d_{lj} is given by

$$d_{lj} \mid \gamma_l, \tau_l, \sigma^2, r \sim \gamma_l \text{ MOM}(\tau_l, \sigma^2, r) + (1-\gamma_l)\ \delta(0),$$

for individual IMOM prior-based analysis, the spike-and-slab prior for the wavelet coefficient d_{lj} is given by

$$d_{lj} \mid \gamma_l, \tau_l, \sigma^2, r \sim \gamma_l \text{ IMOM}(\tau_l, \sigma^2, r) + (1-\gamma_l)\ \delta(0),$$

and for non-local prior mixture-based analysis, the spike-and-slab prior for the wavelet coefficient d_{lj} is given by

$$d_{lj} \mid \gamma_l^{(1)}, \gamma_l^{(2)}, \tau_l^{(1)}, \tau_l^{(2)}, \sigma^2, r, \nu \sim \gamma_l^{(1)} \text{ MOM}(\tau_l^{(1)}, r, \sigma^2) + (1-\gamma_l^{(1)})\gamma_l^{(2)} \text{ IMOM}(\tau_l^{(2)}, \nu, \sigma^2) + (1-\gamma_l^{(1)})(1-\gamma_l^{(2)})\ \delta(0),$$

where the density of the MOM prior is

$$mom(d_{lj}|\tau_l^{(1)}, r, \sigma^2) = \widetilde{M}_r \left(\tau_l^{(1)}\sigma^2\right)^{-r-1/2} d_{lj}^{2r} \exp\left(-\frac{d_{lj}^2}{2\tau_l^{(1)}\sigma^2}\right), \quad r > 1, \tau_l^{(1)} > 0, \widetilde{M}_r = \frac{(2\pi)^{-1/2}}{(2r-1)!!}$$

and the density of the IMOM prior is

$$imom(d_{lj}|\tau_l^{(2)},\nu,\sigma^2) = \frac{\left(\tau_l^{(2)}\sigma^2\right)^{\nu/2}}{\Gamma(\nu/2)}|d_{lj}|^{-\nu-1}\exp\left(-\frac{\tau_l^{(2)}\sigma^2}{d_{lj}^2}\right), \quad \nu>1, \tau_l^{(2)}>0.$$

Hyperparameter specifications:
For non-local prior mixture-based analysis, the available specifications for the mixture probabilities are

1. **Logit:**
$$\gamma_l^{(1)} = \frac{\exp(\theta_1^\gamma - \theta_2^\gamma l)}{1+\exp(\theta_1^\gamma - \theta_2^\gamma l)}, \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{\exp(\theta_3^\gamma - \theta_4^\gamma l)}{1+\exp(\theta_3^\gamma - \theta_4^\gamma l)}, \quad \theta_3^\gamma \in \mathbb{R},\ \theta_4^\gamma > 0$$
2. **Generalized logit or Richards:**
$$\gamma_l^{(1)} = \frac{1}{[1+\exp\{-(\theta_1^\gamma - \theta_2^\gamma l)\}]^{\theta_3^\gamma}}, \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma, \theta_3^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{1}{[1+\exp\{-(\theta_4^\gamma - \theta_5^\gamma l)\}]^{\theta_6^\gamma}}, \quad \theta_4^\gamma \in \mathbb{R},\ \theta_5^\gamma, \theta_6^\gamma > 0;$$
3. **Hyperbolic secant:**
$$\gamma_l^{(1)} = \frac{2}{\pi}\arctan\left[\exp\left(\frac{\pi}{2}\left(\theta_1^\gamma - \theta_2^\gamma l\right)\right)\right], \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{2}{\pi}\arctan\left[\exp\left(\frac{\pi}{2}\left(\theta_3^\gamma - \theta_4^\gamma l\right)\right)\right], \quad \theta_3^\gamma \in \mathbb{R},\ \theta_4^\gamma > 0$$
4. **Generalized normal:**
$$\gamma_l^{(1)} = \frac{1}{2} + \mathrm{sign}(\theta_1^\gamma - l)\frac{1}{2\Gamma(1/\theta_2^\gamma)}\gamma\left(1/\theta_2^\gamma, \left|\frac{\theta_1^\gamma - l}{\theta_3^\gamma}\right|^{\theta_2^\gamma}\right), \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma, \theta_3^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{1}{2} + \mathrm{sign}(\theta_4^\gamma - l)\frac{1}{2\Gamma(1/\theta_5^\gamma)}\gamma\left(1/\theta_5^\gamma, \left|\frac{\theta_4^\gamma - l}{\theta_6^\gamma}\right|^{\theta_5^\gamma}\right), \quad \theta_4^\gamma \in \mathbb{R},\ \theta_5^\gamma, \theta_6^\gamma > 0$$

For individual non-local prior based analysis, $gamma_l$ is defined likewise.
For non-local prior mixture-based analysis, the available specifications for the scale parameters are

1. **Polynomial decay:**
$$\tau_l^{(1)} = \theta_1^\tau l^{-\theta_2^\tau}, \quad \theta_1^\tau, \theta_2^\tau > 0$$
$$\tau_l^{(2)} = \theta_3^\tau l^{-\theta_4^\tau}, \quad \theta_3^\tau, \theta_4^\tau > 0$$
2. **Double-exponential decay:**
$$\tau_l^{(1)} = \theta_1^\tau \exp(-\theta_2^\tau l) + \theta_3^\tau \exp(-\theta_4^\tau l), \quad \theta_1^\tau, \theta_2^\tau, \theta_3^\tau, \theta_4^\tau > 0$$
$$\tau_l^{(2)} = \theta_5^\tau \exp(-\theta_6^\tau l) + \theta_7^\tau \exp(-\theta_8^\tau l), \quad \theta_5^\tau, \theta_6^\tau, \theta_7^\tau, \theta_8^\tau > 0$$

For individual non-local prior based analysis, tau_l is defined likewise.

Note: The wavelet computations are performed by using the R package **wavethresh**.

Value

A list containing the following.

- `data` The data vector.
- `func.post.mean` Posterior estimate (mean) of the function that generated the data.
- `wavelet.empirical` Empirical wavelet coefficients obtained by wavelet transformation of the data.
- `wavelet.post.mean` Posterior estimate (mean) of the true wavelet coefficients obtained by wavelet transformation of the underlying function.
- `hyperparam` Estimates of the hyperparameters that specify the spike-and-slab prior for the wavelet coefficients.
- `sigma` Estimate of the standard deviation of the error.
- `MSE.mean` Mean squared error of the estimates, computable only if true functional values are supplied in the input argument `func`.
- `runtime` System run-time of the function.

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References

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See Also

`wd`, `wr`

Examples

```
# Using the well-known Doppler function to
# illustrate the use of the function BNLPWA

# set seed for reproducibility
set.seed(1)

# Define the doppler function
doppler <- function(x) {
  sqrt(x * (1 - x)) * sin((2 * pi * 1.05) / (x + 0.05))
}

# Generate true values over a grid of length an integer power of 2
n <- 128  # Number of points
```

```
x <- seq(0, 1, length.out = n)
true_signal <- doppler(x)

# Add noise to generate data
sigma <- 0.2  # Noise level
y <- true_signal + rnorm(n, mean = 0, sd = sigma)

# BNLPWA analysis based on MOM prior using logit specification
# for the mixture probabilities and polynomial decay
# specification for the scale parameter
fit_mom <- BNLPWA(data=y, func=true_signal, r=1, wave.family=
  "DaubLeAsymm", filter.number=6, bc="periodic", method="mom",
  mixprob_dist="logit", scale_dist="polynom")

plot(y,type="l",col="grey") # plot of data
lines(fit_mom$func.post.mean,col="blue") # plot of posterior
# smoothed estimates
fit_mom$MSE.mean


  # BNLPWA analysis using non-local prior mixtures using generalized
  # logit (Richard's) specification for the mixture probabilities and
  # double exponential decay specification for the scale parameter
  fit_mixture <- BNLPWA(data=y, func=true_signal, r=1, nu=1, wave.family=
    "DaubLeAsymm", filter.number=6, bc="periodic", method="mixture",
    mixprob_dist="genlogit", scale_dist="doubleexp")

  plot(y,type="l",col="grey") # plot of data
  lines(fit_mixture$func.post.mean,col="blue") # plot of posterior
  # smoothed estimates
  fit_mixture$MSE.mean


# Compare with other wavelet methods
library(wavethresh)
wd <- wd(y, family="DaubLeAsymm", filter.number=6, bc="periodic")  # Wavelet decomposition

wd_thresh_universal <- threshold(wd, policy="universal", type="hard")
fit_universal <- wr(wd_thresh_universal)
MSE_universal <- mean((true_signal-fit_universal)^2)
MSE_universal

wd_thresh_sure <- threshold(wd, policy="sure", type="soft")
fit_sure <- wr(wd_thresh_sure)
MSE_sure <- mean((true_signal-fit_sure)^2)
MSE_sure

wd_thresh_BayesThresh <- threshold(wd, policy="BayesThresh", type="hard")
fit_BayesThresh <- wr(wd_thresh_BayesThresh)
MSE_BayesThresh <- mean((true_signal-fit_BayesThresh)^2)
MSE_BayesThresh

wd_thresh_cv <- threshold(wd, policy="cv", type="hard")
```

```
fit_cv <- wr(wd_thresh_cv)
MSE_cv <- mean((true_signal-fit_cv)^2)
MSE_cv

wd_thresh_fdr <- threshold(wd, policy="fdr", type="hard")
fit_fdr <- wr(wd_thresh_fdr)
MSE_fdr <- mean((true_signal-fit_fdr)^2)
MSE_fdr

# Compare with non-wavelet methods
    # Kernel smoothing
fit_ksmooth <- ksmooth(x, y, kernel="normal", bandwidth=0.05)
MSE_ksmooth <- mean((true_signal-fit_ksmooth$y)^2)
MSE_ksmooth
    # LOESS smoothing
fit_loess <- loess(y ~ x, span=0.1)  # Adjust span for more or less smoothing
MSE_loess <- mean((true_signal-predict(fit_loess))^2)
MSE_loess
    # Cubic spline smoothing
spline_fit <- smooth.spline(x, y, spar=0.5)  # Adjust spar for smoothness
MSE_spline <- mean((true_signal-spline_fit$y)^2)
MSE_spline
```

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